

Date: 21/08/2023

MOODLAKATTE INSTITUTE OF TECHNOLOGY, KUNDAPURA

SECOND SEMESTER B.E – SECOND INTERNAL TEST

NOTICE

1. All Second Semester room invigilators are requested to report at Physics Lab **10 minutes** prior to the test and should reach the allotted test room **05 minutes** before the commencement of IA.

2. Room invigilators has to ensure the count (Answer booklets/ Question papers) while receiving and similar count of **orderly arranged** booklets to be submitted back to the center.

3. Room invigilators should inform the students to search desks, tables, pockets, wallets, instrument boxes and handover if any papers/ notes/ manuscripts/ books or any material such as smart watch, mobile phone, programmable calculators.

Also please check that the students have occupied their respective seats according to seating plan and Instruct students to wear their ID card compulsorily.

4. All Checking Squad Members must visit **at least Four** rooms and shall ensure that no books or any other subject related materials, smart watch, mobile phones carried by students.

Any violations should be brought to the notice of Test Coordinator/ Chief Coordinator.

5. Kindly inform the Test Coordinator in case of any alternatives done to your assigned duties.

6. Room invigilators are requested to ensure the required information is filled by the students and concerned invigilators should acknowledge the same with their signature on the starting page of answer booklet for every internal.



Mr. Balanageshwar

FY Test Coordinator

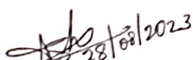


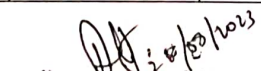
HOD of Basic Science & Humanities
Mr. Deenak Srinivas
Moodlakatte Institute of Technology
Kundapura - 576217
Moodlakatte, Basic Science and humanities.

MOODLAKATTE INSTITUTE OF TECHNOLOGY, KUNDAPURA.
DEPARTMENT OF BASIC SCIENCE AND HUMANITIES

TIME TABLE
II INTERNAL ASSESSMENT TEST EVEN SEM

DATE	TIME	SEMESTER : II		
		SECTION A	SECTION B	SECTION C
4-Sep-23	09.30 AM TO 10.30 AM	MATHEMATICS FOR CSE STREAM II (BMATS201)	MATHEMATICS FOR CSE STREAM II (BMATS201)	MATHEMATICS FOR EES-II (BMATE201)
4-Sep-23	10.30 AM TO 11.00 AM	PROFESSIONAL WRITING SKILLS IN ENGLISH (BPWSK206)	PROFESSIONAL WRITING SKILLS IN ENGLISH (BPWSK206)	PROFESSIONAL WRITING SKILLS IN ENGLISH (BPWSK206)
5-Sep-23	09.30 AM TO 10.30 AM	CHEMISTRY FOR CSE STREAM (BCHES202)	PHYSICS FOR CSE STREAM (BPHYS202)	PHYSICS FOR EES (BPHYE202)
5-Sep-23	10.30 AM TO 11.00 AM	SAMSKRUTIKA KANNADA/BALAKE KANNADA (BKSKK207/BKBKK207)	SAMSKRUTIKA KANNADA/BALAKE KANNADA (BKSKK207/BKBKK207)	SAMSKRUTIKA KANNADA/BALAKE KANNADA (BKSKK207/BKBKK207)
6-Sep-23	09.30 AM TO 10.30 AM	INTRODUCTION TO C++ PROGRAMMING (BPLCK205D)	INTRODUCTION TO WEB PROGRAMMING (BPLCK205A)	INTRODUCTION TO C++ PROGRAMMING (BPLCK205D)
6-Sep-23	10.30 AM TO 11.00 AM	SCIENTIFIC FOUNDATION OF HEALTH (BSFHK258)	SCIENTIFIC FOUNDATION OF HEALTH (BSFHK258)	SCIENTIFIC FOUNDATION OF HEALTH (BSFHK258)
7-Sep-23	09.30 AM TO 10.30 AM	INTRODUCTION TO ELECTRONICS ENGINEERING (BESCK204C)	INTRODUCTION TO ELECTRONICS ENGINEERING (BESCK204C)	INTRODUCTION TO ELECTRICAL ENGINEERING (BESCK204B)
8-Sep-23	09.30 AM TO 10.30 AM	-----	PRINCIPLES OF PROGRAMMING USING C (BPOPS203)	BASIC ELECTRONICS (BBEE203)


I.A. CO-ORDINATOR


Basic Science & Humanities
Moodlakatte Institute of Technology,
Moodlakatte, Kundapur 576 217
Udipi Dist. Karnataka


PRINCIPAL

MOODLAKATTE INSTITUTE OF TECHNOLOGY, KUNDAPURA



SECOND SEMESTER B.E.

II INTERNAL DUTY CHART.

DATE / SESSION	04/09/2023	05/09/2023	06/09/2023	07/09/2023	08/09/2023
	Morning	Morning	Morning	Morning	Morning
Lh. No 211	Akshatha (BSH)	Chaithra (BSH)	Shrinidhi (CS)	Vishwanath (Mech)	-----
Lh. No 212	Smithashri (CS)	Subramanya (BSH)	Farana (CS)	Vignesh (BSH)	Subramanya (BSH)
Lh. No 213	Vishwanath Shervegar (EC)	Prameela (CS)	Nikhila (BSH)	Akshatha (BSH)	Chaithra (BSH)

Note

1. If the faculty is not able to perform the duty, he/she should make alternate arrangement for the same

2. The faculty should collect QP, B form etc 10 minutes before the commencement of the IA exam in BSH HOD chamber.


31/8/2023

Basid HOD
Moodlakatte Institute of Technology
Moodlakatte, Kundapur - 576 217
Kannur Dist. Karnataka



Moodlakatte Institute of technology, kundapura.

Seating arrangement for SECOND Internal Assesment FROM 4 TO 8 OF SEP 2023

Sl. No.	Year/ Branch	1	2	3	4
		Lh. No. 211	Lh. No. 212	Lh. No. 213	LIB
		40	32	14+16C	
1	Ist B.E A sec	(1-42)	(43-74)	(75-80)	
2	Ist B.E B sec	(63-82)	(31-62)	(1-30)	
3	Ist B.E C sec	(29-42)		(22-28)	(2-21)

NOTE: The Students should present in the classroom 15min before the commencement of the IA exam .

MOODLAKATTE INSTITUTE OF TECHNOLOGY, KUNDAPURA.

B FORM

YEAR: 1 SEC 'A'

BRANCH: AI, CS, DS

Room No. : 211

IA TEST: II

[illegible]

	TOTAL NO OF STUDENTS	40	40	40	40	40	40	40
	TOTAL NO OF ABSENTEES	04	04	01	01	01	01	01
	TOTAL NO OF PRESENTEES	39	39	39	39	39	39	39
SIGNATURE OF	ROOM SUPERINTENDENT	J. S. S.	J. S. S.	A	A	A	A	B. S.
	EXAM COORDINATOR	A	A	A	A	A	A	A
	SUBJECT INCHARGE	J. S. S.	A	A	A	A	A	A

MOODLAKATTE INSTITUTE OF TECHNOLOGY, KUNDAPURA.

B FORM

YEAR: 1 SEC 'A'

BRANCH: AI, CS, DS Room No. : 212

IA TEST: II

[illegible][illegible]

MOODLAKATTE INSTITUTE OF TECHNOLOGY, KUNDAPURA.

B FORM

YEAR: 1 SEC 'A'

BRANCH: AI, CS, DS

Room No. : 213

IA TEST: II

ROLL NO.	NAME OF THE PUPIL	SIGNATURE OF THE PUPIL AT EXAMINATION SESSIONS						
		04/09/2023	04/09/2023	05/09/2023	05/09/2023	06/09/2023	06/09/2023	07/09/2023
		Morning	Morning	Morning	Morning	Morning	Morning	Morning
75	Trupti Gajanan Naik							
76	Vaishnavi	vaishnavi	vaishnavi	vaishnavi	vaishnavi	vaishnavi	vaishnavi	vaishnavi
77	Vaishnavi Mendon J	vaishnavi	vaishnavi	vaishnavi	vaishnavi	vaishnavi	vaishnavi	vaishnavi
78	Venisha Braganza	-AB-	A-B-					
79	Vinay U							
80	Yashaswini Naik	-AB-	A-B-	Yashaswini	Yashaswini	Yashaswini	Yashaswini	Yashaswini

[illegible]

MOODLAKATTE INSTITUTE OF TECHNOLOGY, KUNDAPURA.

B FORM

YEAR: 1 SEC 'B'

BRANCH: AI, CS, DS Room No. : 213

IA TEST: II

[illegible]

TOTAL NO OF STUDENTS	30	30	30	30	30	30	30	30
TOTAL NO OF ABSENTEES	01	01	0	0	01	01	01	02
TOTAL NO OF PRESENTEES	29	29	30	30	29	29	29	28
ROOM SUPERINTENDENT	H	H	H	H	M.D.	M.D.	H	Chaudhary
EXAM COORDINATOR	Jal	H	H	H	H	H	H	Jal
SUBJECT INCHARGE	Deepti Parvate	Sinhrao			A	M	X	Mus

SIGNAT
URE OF

MOODLAKATTE INSTITUTE OF TECHNOLOGY, KUNDAPURA.

B FORM

YEAR: 1 SEC 'B'

BRANCH: AI, CS, DS Room No. : 211 IA TEST: II

IA TEST: II

[illegible]

	TOTAL NO OF STUDENTS	20	20	20	20	20	20	20	20
	TOTAL NO OF ABSENTEES	01	01	01	01	—	—	—	—
	TOTAL NO OF PRESENTEES	19	19	19	19	20	20	20	20
SIGNATURE OF	ROOM SUPERINTENDENT	Joddy	Joddy	A	A	S	S	A	A
	EXAM COORDINATOR	A	A	A	A	A	A	A	A
	SUBJECT INCHARGE	<u>Deepal</u>	<u>Purany</u>	<u>Sudana</u>	A	<u>P</u>	<u>Chasth</u>	A	<u>C</u>

MOODLAKATTE INSTITUTE OF TECHNOLOGY, KUNDAPURA.

B FORM

YEAR: 1 SEC 'C'

BRANCH: EC

Room No. : LIB

IA TEST: II

[illegible]

	TOTAL NO OF STUDENTS	20	20	20	20	20	20	20	20
	TOTAL NO OF ABSENTEES	—	—	0	0	0	0	01	0
	TOTAL NO OF PRESENTEES	20	20	20	20	20	20	19	20
SIGNATURE OF	ROOM SUPERINTENDENT	<i>[Signature]</i>	<i>[Signature]</i>	<i>[Signature]</i>	<i>[Signature]</i>	<i>[Signature]</i>	<i>[Signature]</i>	<i>[Signature]</i>	<i>[Signature]</i>
	EXAM COORDINATOR	<i>[Signature]</i>	<i>[Signature]</i>	<i>[Signature]</i>	<i>[Signature]</i>	<i>[Signature]</i>	<i>[Signature]</i>	<i>[Signature]</i>	<i>[Signature]</i>
	SUBJECT INCHARGE	<i>Charli</i>	<i>Pavani</i>	<i>Sudhansu</i>	<i>[Signature]</i>	<i>[Signature]</i>	<i>[Signature]</i>	<i>[Signature]</i>	<i>[Signature]</i>



MOODLAKATTE INSTITUTE OF TECHNOLOGY, KUNDAPURA.

B FORM

YEAR: 1 SEC 'C'

BRANCH: EC

Room No. : 213

IA TEST: II

ROLL NO.	NAME OF THE PUPIL	SIGNATURE OF THE PUPIL AT EXAMINATION SESSIONS							
		04/09/2023	04/09/2023	05/09/2023	05/09/2023	06/09/2023	06/09/2023	07/09/2023	08/09/2023
		Morning	Morning	Morning	Morning	Morning	Morning	Morning	Morning
22	Manvith Devadiga	<u>Manvith</u>	<u>manvith</u>	<u>Manvith</u>	<u>Manvith</u>	<u>Manvith</u>	<u>Manvith</u>	<u>Manvith</u>	<u>Manvith</u>
23	Maruti Madlur	<u>Maruti</u>	<u>Maruti</u>	<u>Maruti</u>	<u>Maruti</u>	<u>Maruti</u>	<u>Maruti</u>	<u>Maruti</u>	<u>Maruti</u>
24	Mithun Govinda poojari	<u>Mithun</u>	<u>Mithun</u>	<u>Mithun</u>	<u>Mithun</u>	<u>Mithun</u>	<u>Mithun</u>	<u>Mithun</u>	<u>Mithun</u>
25	Mohammed Rihan	<u>Rihan</u>	<u>Rihan</u>	<u>Rihan</u>	<u>Rihan</u>	<u>Rihan</u>	<u>Rihan</u>	<u>Rihan</u>	<u>Rihan</u>
26	Naganagouda B Soratur	<u>NB</u>	<u>NB</u>	<u>NB</u>	<u>NB</u>	<u>NB</u>	<u>NB</u>	<u>NB</u>	<u>NB</u>
27	Nanditha	<u>Nanditha</u>	<u>Nanditha</u>	<u>Nanditha</u>	<u>Nanditha</u>	<u>Nanditha</u>	<u>Nanditha</u>	<u>Nanditha</u>	<u>Nanditha</u>
28	Nikhil	<u>Nikhil</u>	<u>Nikhil</u>	<u>Nikhil</u>	<u>Nikhil</u>	<u>Nikhil</u>	<u>Nikhil</u>	<u>Nikhil</u>	<u>Nikhil</u>

SIGNATURE OF	TOTAL NO OF STUDENTS	7	7	7	7	7	7	7
	TOTAL NO OF ABSENTEES	0	0	0	0	0	0	0
	TOTAL NO OF PRESENTEES	7	7	7	7	7	7	7
	ROOM SUPERINTENDENT	<u>Chaitu</u>	<u>Parang</u>	<u>Subram</u>	<u>Subram</u>	<u>Subram</u>	<u>Subram</u>	<u>Subram</u>
	EXAM COORDINATOR	<u>Chaitu</u>	<u>Parang</u>	<u>Subram</u>	<u>Subram</u>	<u>Subram</u>	<u>Subram</u>	<u>Subram</u>



Moodlakatte Institute of Technology, Kundapura

Kundapura

Department of Basic Science

II - Internal Assessment

Semester: 2-CBCS 2022

Subject: MATHEMATICS FOR EES-II (BMATE201)

Faculty: Ms Chaithra

Date: 4 Sep 2023

Time: 09:30 AM - 10:30 AM

Max Marks: 25

Answer any 2 question(s)

Answer any 2 question(s)																				
Q.No			Marks	CO	PO	BT/CL														
1	a	Find a real root of the equation $x^3 - 4x - 9 = 0$ correct to three decimal places by the method of false position in (2,3)	6	CO4	PO3	L3														
	b	Construct Newton's forward interpolation polynomial for the data <table><tr><td>x</td><td>4</td><td>6</td><td>8</td><td>10</td></tr><tr><td>f(x)</td><td>1</td><td>3</td><td>8</td><td>16</td></tr></table>	x	4	6	8	10	f(x)	1	3	8	16	7	CO4	PO2	L4				
x	4	6	8	10																
f(x)	1	3	8	16																
OR																				
2	a	Use Taylor series method to find y(0.1) from $\frac{dy}{dx} = e^x - y^2$, with y(0)=1	6	CO4	PO2	L4														
	b	Apply Milne's method to find y(0.8) given $\frac{dy}{dx} + xy^2 = 0$ <table><tr><td>x</td><td>0</td><td>0.2</td><td>0.4</td><td>0.6</td></tr><tr><td>y</td><td>2</td><td>1.9231</td><td>1.7214</td><td>1.4706</td></tr></table>	x	0	0.2	0.4	0.6	y	2	1.9231	1.7214	1.4706	7	CO4	PO2	L5				
x	0	0.2	0.4	0.6																
y	2	1.9231	1.7214	1.4706																
3	a	Evaluate $\int_0^3 \frac{x}{\cos x} dx$ by using Simopson's 3/8 th rule,by taking 6 equal intervals(x is in radians).	6	CO4	PO2	L3														
	b	Using Newton's forward difference formula,find f(38) <table><tr><td>x</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td></tr><tr><td>y</td><td>184</td><td>204</td><td>226</td><td>250</td><td>276</td><td>304</td></tr></table>	x	40	50	60	70	80	90	y	184	204	226	250	276	304	6	CO4	PO2	L4
x	40	50	60	70	80	90														
y	184	204	226	250	276	304														

OR					
4	a	Using Modified Euler's method to find y at x=0.2 given $\frac{dy}{dx} = x - y^2$ and y(0)=1 by taking step size h=0.1	6	CO4	PO1 L3
	b	Given $\frac{dy}{dx} = 3x + \frac{y}{2}$, y(0)=1. Compute y(0.2) by taking h=0.2 using Runge-Kutta method of fourth order.	6	CO4	PO2 L5

CO4 : Apply the knowledge of numerical methods in solving physical and engineering phenomena.

Approved

HOD of Basic Science & Humanities
Moodlakatte Institute of Technology,
Moodlakatte, Kundapura - 576217

1. Find a real root of equations $x^3 - 4x - 9 = 0$, correct to three decimal place by the method of false position in (2,3)

$$\text{Given} = x^3 - 4x - 9 = 0$$

$$\Rightarrow x_0 = 2 \quad x_1 = 3$$

$$f(x) = x^3 - 4x - 9$$

} — (1)

$$x_2 = x_0 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} \times f(x_0)$$

$$= 2 - \frac{(3 - 2)}{6 + 9} (-9)$$

$$= \underline{2.6}$$

$$f(x_2) = f(2.6) = 1.824$$

———— (1)

$$x_3 = 2.6 - \frac{(3 - 2.6)}{6 + 1.824} \times (-1.824)$$

$$= \underline{2.6932}$$

$$f(x_3) = f(2.6932) = -0.2381$$

———— (1)

$$x_4 = 2.6932 - \frac{(3 - 2.6932)}{(6 + 0.2381)} \times (-0.2381)$$

$$= \underline{2.7049}$$

———— (1)

$$f(x_4) = f(2.7049) = -0.0292$$

$$x_5 = 2.7049 - \frac{(3 - 2.7049)}{(6 + 0.0292)} \times (-0.0292)$$

$$x_5 = \underline{2.7063}$$

$$f(x_5) = f(2.7063) = -0.0040$$

———— (1)

$$x_5 = 2.7063 - \frac{(3 - 2.7063)}{6 + 0.0040} \times (-0.0040)$$

$$x_5 = \underline{2.706}$$

$$f(x_6) = -0.0004$$

①

The approximate root eqⁿ of regula-Falsi method is 2.706.

1b. Construct Newton's forward interpolation polynomial

-1 for the data

x	4	6	8	10
$f(x)$	1	3	8	16

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
4	1	2	3	0
6	3	5	3	
8	8	8		
10	16			

②

Then we have Newton-forward interpolation formula.

$$y_n = y_0 + \frac{n}{1} \Delta y_0 + \frac{n(n-1)}{2!} \Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!} \Delta^4 y_0 + \dots$$

$$\Delta^4 y_0 + \dots$$

③

Q. Use Taylor Series method to find $y(0.1)$ from $\frac{dy}{dx} = e^x - y^2$ with $y(0) = 1$

Soln: Given $\frac{dy}{dx} = e^x - y^2 = y'(x) \rightarrow \text{---} \textcircled{1}$

And $y(0) = 1 \Rightarrow x_0 = 0, y_0 = 1$

$\therefore y'(x_0) = e^{x_0} - y_0^2 = e^0 - 1^2 = 1 - 1 = 0$

$\therefore y''(x) = e^x - 2yy'$ $\text{---} \textcircled{1}$

$y''(x_0) = e^{x_0} - 2y_0y'_0 = e^0 - 2(1)(0) = 1$

$\therefore y'''(x) = e^x - 2[y'y' + yy'']$

$y'''(x_0) = e^{x_0} - 2[y'y' + y_0y_0'']$
 $= e^0 - 2[0 + (1)(1)] = 1 - 2 = -1$ $\text{---} \textcircled{1}$

$y^{(4)}(x) = e^x - 2[2y'y'' + y'y''' + yy''']$

$y^{(4)}(x) = e^x - 2[3y'y'' + yy''']$

$y^{(4)}(x_0) = e^{x_0} - 2[3y_0'y_0'' + y_0y_0''']$

$y^{(4)}(x_0) = e^0 - 2[3(0)(1) + (1)(-1)]$

$1 + 2$
 $y^{(4)}(x_0) = 1 + 2 = 3$ $\text{---} \textcircled{1}$

WKT,

$y(x) = y_0 + \frac{(x-x_0)^1}{1!} y'(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0) + \frac{(x-x_0)^3}{3!} y'''(x_0) + \dots$ $\text{---} \textcircled{1}$

$= y(x) = 1 + \frac{x}{1!}(0) + \frac{x^2}{2!}(1) + \frac{x^3}{3!}(-1) + \frac{x^4}{4!}(3) + \dots$

$y(x) = 1 + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{8} + \dots$

$\therefore y(0.8) = 1 + \frac{(0.8)^2}{2} - \frac{(0.8)^3}{6} + \frac{1}{8}(0.8)^4 = \underline{1.089}$ $\text{---} \textcircled{1}$

Qb. Apply Milne's method to find $y(0.8)$ given $\frac{dy}{dx} + xy^2 = 0$.

x	0	0.2	0.4	0.6
y	2	1.9831	1.7814	1.4706

Soln Given $\frac{dy}{dx} = -xy^2$

$\Rightarrow$ we prepared the following table using given data which is essentially required for apply the predictor-corrector formula.

x	y	$y' = -xy^2$
$x_0 = 0$	2	0
$x_1 = 0.2$	1.983	-0.7395
$x_2 = 0.4$	1.7814	-1.1852
$x_3 = 0.6$	1.4706	-1.2976
$x_4 = 0.8$	$y_4 = ?$	

Predictor & corrector formula,

$$y_4^{(p)} = y_0 + \frac{4h}{3} [2y'_1 - y'_2 + y'_3]$$

$$2 + \left(\frac{4 \times 0.2}{3}\right) [2 \times (-0.7395) - 1.1852 + 2 \times (-1.2976)]$$

$$= y_4^{(p)} = -2.8893$$

$$\text{Now } y_4^{(1)} = -x_4 (y_4^{(p)})^2 = -1.209$$

Now we have,

$$y_4^{(c)} = y_3 + \frac{h}{3} [y'_3 + 4y'_4 + y'_5]$$

$$= 1.7814 + \frac{0.2}{3} [-1.1853 + 4(-1.2976) - 1.20938]$$

$$y_4^{(c)} = -1.585$$

$$y_4^{(c)} = -x_4 [y_4^{(c)}]^2 = (0.8)^2 \times (1.5145)^2 = -1.1800$$

$$y_4^{(c)} = 1.7814 + (0.2/3)(-1.1853 - 4 \times -1.2976 - 1.209) = 1.814 = y(0.8)$$

Q. Evaluate $\int_0^3 \frac{x}{\cos x} dx$ by using Simpson's $3/8$ th rule. by taking 6 equal intervals (x is in radians)

Soln:- Given: $I = \int_0^3 \frac{x}{\cos x}$

where, $a=0$, $b=3$, $n=6$

$\therefore h = \frac{b-a}{n} = \frac{3-0}{6} = \frac{1}{2} = 0.5$

Then we have value of $x: a = x_0 = 0$.

$x_1 = x_0 + h = 0 + 0.5 = 0.5$

$x_2 = x_1 + h = 0.5 + 0.5 = 1$

$x_3 = x_2 + h = 1 + 0.5 = 1.5$

$x_2 = x_1 + h = 0.5 + 0.5 = 1$

$x_4 = x_3 + h = 1.5 + 0.5 = 2$

$x_6 = x_5 + h = 2.5 + 0.5 = 3 = b$

Then we have $f(x) = \frac{x}{\cos x}$

at $x=0$, $f(x_0) = 0 = y_0$

at $x=1$, $f(x_1) = 0.56 = y_1$

at $x=2$, $f(x_2) = 1.85 = y_2$

at $x=3$, $f(x_3) = 21.20 = y_3$

at $x=4$, $f(x_4) = -4.80 = y_4$

at $x=5$, $f(x_5) = -3.120 = y_5$

at $x=6$, $f(x_6) = -3.03 = y_6$

we have Simpson's $(1/8)^{th}$ rule

$I = \frac{3h}{8} [(y_0 + y_6) + 2(y_3) + 3(y_1 + y_2 + y_4 + y_5)]$

$$I = \frac{3 \times 0.5}{8} [(3.03) + 2(21.20) + 3(0.56 + 1.85 - 4.80 - 3.120)]$$

$$= \frac{3 \times 0.5}{8} [22.84] = \frac{22.84 \times 1.5}{8}$$

$$= \underline{\underline{4.2855}}$$

3b. Using Newton's forward difference formula, find

$f(38)$

x	40	50	60	70	80	90
y	184	204	226	250	276	304

Soln :-

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	184	20	2	0	
50	204	22	2	0	
60	226	24	2	0	
70	250	26	2		
80	276	28			
90	304				

$$\text{Now } u = \frac{x - x_0}{h} = \frac{38 - 40}{10} = -0.2 \quad \text{--- (1)}$$

Then we have Newton forward interpolation formula.

$$y_u = y_0 + u \Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^3 y_0 + \dots$$

--- (1)

$$= y_1 = 184 + (-0.2)(20) + (-0.2) \frac{(-0.2-1)}{2} (8) + \frac{(-0.2)(0.2-1)(0.2-2)(0)}{6} \quad \text{--- (1)}$$

$$= y_1 = 180 + 0.24$$

$$= f(38) = \underline{\underline{180.24}} \quad \text{--- (1)}$$

4a. Using Modified Euler's method to find y at $x=0.2$
 given $\frac{dy}{dx} = x-y^2$ and $y(0)=1$ by taking step size $h=0.1$

Soln:-

$$\text{Given } \frac{dy}{dx} = x^2 - y^2 = f(x, y)$$

$$\text{and } y(0)=1 \Rightarrow x_0=0, y_0=1, h=0.1$$

To find $y(x_1)=y_1$:-

$$\therefore y(x_0+h) = y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + 0.1(0, 1)$$

$$y_1^{(0)} = \underline{\underline{0.9}} \quad \text{--- (1)}$$

By modified Euler method,

$$y_1^{(1)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(0)})]$$

$$= 1 + \frac{0.1}{2} [f(0, 1) + f(0.1, 0.9)]$$

$$y_1^{(1)} = \underline{\underline{0.9145}}$$

$$y_1^{(2)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(1)})]$$

$$= 1 + \frac{0.1}{2} [(0, 1) + (0.1, 0.9145)]$$

$$= \underline{\underline{0.9132}} \quad \text{--- (1)}$$

$$y_1^{(3)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(2)})]$$

$$= 1 + \frac{0.1}{2} [(0, 1) + (0.1, 0.9132)]$$

$$= \underline{0.9133}$$

$$\therefore y(0.1) = \underline{0.9133}$$

————— (1)

To find $y(x_2) = y_2$:-

By Euler formula, $y_2^{(0)} = y_0 + hf(x_0, y_0)$

$$= y_2^{(0)} = 0.9133 + (0.2)f(0.1, 0.9133)$$

$$= y_2^{(0)} = \underline{0.7665}$$

—————

By modified Euler

$$y_2^{(1)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_2^{(0)})]$$

$$= 0.9133 + \frac{0.2}{2} [f(0.1, 0.9133) + f(0.2, 0.7665)]$$

$$= 0.9133 + \frac{0.2}{2} [(0.1, 0.9133) + (0.2, 0.7665)]$$

$$y_2^{(1)} = \underline{0.8012}$$

————— (1)

$$y_2^{(2)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_2^{(1)})]$$

$$= 0.9133 + \frac{0.2}{2} [f(0.1, 0.9133) + (0.1, 0.8012)]$$

$$= 0.7957$$

$$y_3^{(3)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_2^{(2)})]$$

————— (1)

$$= 0.9133 + \frac{0.2}{2} [f(0.1, 0.9133) + (0.1, 0.7957)]$$

$$y_1^{(3)} = \underline{0.7966} \Rightarrow y(0.2) = \underline{0.7966}$$

4. Given $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$. Compute $y(0.8)$ by taking $h = 0.2$ using Runge-Kutta method of fourth order.

Given: $\frac{dy}{dx} = 3x + \frac{y}{2} = f(x, y)$

$y(0.1) = 1 \Rightarrow x_0 = 0, y_0 = 1, h = 0.2$ ————— ①

$\therefore K_1 = hf(x_0, y_0) = (0.2)f(0, 1)$
 $= (0.2)(0.5)$ ————— ①

$\Rightarrow K_1 = 0.10$

$\therefore K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$= 0.2f(0.1, 1.05)$

$= 0.2(0.8250)$ ————— ①

$\Rightarrow K_2 = 0.1650$

$\therefore K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$

$= 0.2f(0.1, 1.0825)$

$= 0.2(0.8413)$ ————— ①

$\Rightarrow K_3 = 0.1683$

$\therefore K_4 = hf(x_0 + h, y_0 + K_3)$

$= 0.2f(0.2, 1.1683)$

$K_4 = 0.2368$ ————— ①

WKT,

$y(x) = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$y(0.8) = 1 + \frac{1}{6}[0.1 + 0.3300 + 0.3366 + 0.4736]$

$= 1 + \frac{1}{6}[0.9434]$ ————— ①

$= y(0.8) = \underline{\underline{1.1672}}$

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MOODLAKATTE INSTITUTE OF TECHNOLOGY

MOODLAKATTE-576 217, **KUNDAPURA** DIST : UDUPI

(Affiliated to VTU Belagavi and Approved by AICTE New Delhi)

INTERNAL ASSESSMENT BOOK

ACADEMIC YEAR 2022 -20 23

Department/Programme:	Basic Science And Humanities		
Subject Title:	Engineering Mathematics II - ECE stream	Subject Code:	BMATE201

Name: Lavanya . K Roll No.: 15

USN:

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 SEM: IInd Section: C

FOR STAFF USE ONLY

IA TEST	Date	Invigilator's Sign.	Marks Obtained	Subject Faculty in-charge signature	Average of 3 IA Test	Final Assignment Marks
			Max.Marks (30)			
I	10/07/23		20/25		12/15	10/10
II	04/09/23		20 1/2/25			
III						

Final CIE Marks (IA Test + Assignment): _____

17
50

33
25

50
50

Signature of Student

Signature of Subject Faculty in-charge

Signature of
Basic Science and Humanities
Moodlakatte Institute of Technology
Moodlakatte, Kundapur - 576217
Shri Shakhambhari Printers, Udupi Dist Karnataka

I - Internal Assessment

Q 1)

Given,

$$\phi = x^2 y z + 4 x z^2, \quad (1, 2, -1) \text{ along } (2i - j - 2k)$$

We have Directional Derivative $\Rightarrow \nabla \phi \cdot n$

We know,

$$\nabla = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z}$$

then,

$$\nabla \phi = i \frac{\partial}{\partial x} (x^2 y z + 4 x z^2) + j \frac{\partial}{\partial y} (x^2 y z + 4 x z^2) + k \frac{\partial}{\partial z} (x^2 y z + 4 x z^2)$$

$$\nabla \phi = i(2xy z + 4z^2) + j(x^2 z) + k(x^2 y + 8xz)$$

at $(1, 2, -1)$

$$\left. \nabla \phi \right|_{(1, 2, -1)}$$

$$= i(2(1)(2)(-1) + 4(-1)^2) + j(1^2(-1)) + k(1^2(2) + 8(1)(-1))$$

$$= i(-4 + 4) + j(-1) + k(2 - 8)$$

$$\therefore \nabla \phi = -j - 6k$$

Now,

$$\text{Unit vector, } n = \frac{\text{grad } f}{|\text{grad } f|} = \frac{2i - j - 2k}{\sqrt{4 + 1 + 4}} = \frac{2i - j - 2k}{3}$$

then,

$$\text{Directional Derivative, } \nabla \phi \cdot n = -j - 6k \cdot \left(\frac{2i - j - 2k}{3} \right)$$

$$\therefore \nabla \phi \cdot n = \frac{1}{3} (1 + 12) = \frac{13}{3}$$

Assignment - I

1) b)

given,

$$\vec{F} = (axy + z^3)\vec{i} + (3x^2 - 2)\vec{j} + (bxz^2 - y)\vec{k}.$$

the given $\vec{F}$ is irrotational, i.e., $\text{div} \cdot \vec{F} = 0$

now,

$$\text{div} \cdot \vec{F} \Rightarrow \nabla \cdot \vec{F} = \frac{\partial}{\partial x} F_1 + \frac{\partial}{\partial y} F_2 + \frac{\partial}{\partial z} F_3 = 0$$

$$\Rightarrow \frac{\partial}{\partial x} (axy + z^3) + \frac{\partial}{\partial y} (3x^2 - 2) + \frac{\partial}{\partial z} (bxz^2 - y) = 0$$

$$\Rightarrow ay + 0 + 2bxz = 0$$

Now,

$$F = \nabla \phi$$

$$x\vec{i} - \vec{j} - \vec{k} = \nabla \phi$$

$$\frac{x\vec{i} - \vec{j} - \vec{k}}{\sqrt{1+1+1}} = \frac{\nabla \phi}{|\nabla \phi|} = n \text{, where } n \text{ is a scalar function}$$

$$\left(\frac{x}{\sqrt{3}}\vec{i} - \frac{1}{\sqrt{3}}\vec{j} - \frac{1}{\sqrt{3}}\vec{k} \right) \cdot \nabla \phi = n \cdot \nabla \phi$$

$$\frac{1}{\sqrt{3}} = n \cdot \nabla \phi$$

3) a) Given, $\vec{A} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$

then we know,

Solenoidal is $\text{div} \cdot \vec{F} \Rightarrow \nabla \cdot \vec{F} = 0$

So, and Here, $F_1 = y^2 - z^2 + 3yz - 2x$, $F_2 = 3xz + 2xy$, $F_3 = 3xy - 2xz + 2z$

$$\begin{aligned} \text{div} \cdot \vec{F} = \nabla \cdot \vec{F} &= \frac{\partial}{\partial x}(F_1) + \frac{\partial}{\partial y}(F_2) + \frac{\partial}{\partial z}(F_3) \\ &= \frac{\partial}{\partial x}(y^2 - z^2 + 3yz - 2x) + \frac{\partial}{\partial y}(3xz + 2xy) + \frac{\partial}{\partial z}(3xy - 2xz + 2z) \\ &= -2 + 2x + -2x + 2 \end{aligned}$$

$$\therefore \text{div} \cdot \vec{F} = 0$$

Hence $\vec{A}$ is Solenoidal.

now,

Irrationalal is $\text{curl} \times \vec{F} \Rightarrow \nabla \times \vec{F} = 0$

$$\text{curl} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$\text{curl} \times \vec{F} = \vec{i} \left(\frac{\partial}{\partial y}(3xy - 2xz + 2z) - \frac{\partial}{\partial z}(3xz + 2xy) \right) - \vec{j} \left(\frac{\partial}{\partial x}(3xy - 2xz + 2z) - \frac{\partial}{\partial z}(y^2 - z^2 + 3yz - 2x) \right) + \vec{k} \left(\frac{\partial}{\partial x}(3xz + 2xy) - \frac{\partial}{\partial y}(y^2 - z^2 + 3yz - 2x) \right)$$

$$= \vec{i} (3x - 2x) - \vec{j} (3y + 2z - (-2z) + 3y) + \vec{k} (2y + 2y - 2y)$$

$$\text{curl} \times \vec{F} = \vec{i} (3x - 3x) - \vec{j} (3y + 2z - (-2z) + 3y) + \vec{k} (2y + 2y - 2y)$$

$$\text{curl } \vec{F} = \hat{i}(3x - 3x) - \hat{j}(3y + 2z + 2z + 3y) + \hat{k}(3z + 2y - (2y + 3z))$$

$$\text{curl } \vec{F} = \hat{i}(3x - 3x) - \hat{j}(3y + 2z + 2z + 3y) + \hat{k}(3z + 2y - 2y - 3z)$$

$$\therefore \text{curl } \vec{F} = \vec{0}$$

Hence $\vec{A}$ is irrotational.

3) b) i) $e^{-3t} (2 \cos 5t - 3 \sin 5t)$

$$\Rightarrow L\{e^{at} \cdot f(t)\} = -i \frac{d}{ds} (f(s-a))$$

$$L\{f(t)\} = L\{2 \cos 5t - 3 \sin 5t\}$$

$$= L\{2 \cos 5t\} - L\{3 \sin 5t\}$$

$$= 2L\{\cos 5t\} - 3L\{\sin 5t\}$$

$$= 2 \cdot \frac{s}{s^2 + 5^2} - 3 \cdot \frac{5}{s^2 + 5^2}$$

$$= \frac{2s}{s^2 + 5^2} - \frac{15}{s^2 + 5^2}$$

$$= \frac{2s - 15}{(s^2 + 5^2)} = f(s)$$

Now,

$$L\{e^{at} \cdot f(t)\} = -i \frac{d}{ds} (f(s-a))$$

$$= -1 \frac{d}{ds} = \left(\frac{2s-15}{(s^2+5^2)^2} \right)_{s \rightarrow s+3}$$

$$= \frac{2(s+3)-15}{((s+3)^2 + 5^2)^2}$$

$$= \frac{2(s+3)-15}{(s^2+9+25)^2} = \frac{2s+6-30}{(s^2+9+25)^2} = \frac{2s-24}{(s^2+34)^2}$$

ii) $\frac{\cos at - \cos bt}{x}$

$$\rightarrow \mathcal{L} \left\{ \frac{f(t)}{x} \right\} = \int_s^\infty f(s)$$

$$\mathcal{L} \{ f(t) \} = \mathcal{L} \{ \cos at - \cos bt \}$$

$$= \mathcal{L} \{ \cos at \} - \mathcal{L} \{ \cos bt \}$$

$$= \frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}$$

Now,

$$\mathcal{L} \left\{ \frac{f(t)}{x} \right\} = \mathcal{L} \left\{ \frac{s}{s^2+a^2} - \frac{s}{s^2+b^2} \right\}$$

$$= \mathcal{L} \left\{ \frac{s}{s^2+a^2} \right\} - \mathcal{L} \left\{ \frac{s}{s^2+b^2} \right\}$$

$$= \frac{1}{2} \log(s^2+a^2) - \frac{1}{2} \log(s^2+b^2)$$

$$= \frac{1}{2} \left[\log \frac{s^2 + a^2}{s^2 + b^2} \right]$$

$$= \lim_{s \rightarrow \infty} \frac{1}{2} \left(\log \frac{s^2 + a^2}{s^2 + b^2} \right)$$

$$= \frac{1}{2} \left[\log \left(\frac{1 + \frac{a^2}{s^2}}{1 + \frac{b^2}{s^2}} \right) \right] - \log \left(\frac{s^2 + a^2}{s^2 + b^2} \right)$$

$$= -\frac{1}{2} \log \left(\frac{s^2 + a^2}{s^2 + b^2} \right)$$

$$= \frac{1}{2} \log \left(\frac{s^2 + b^2}{s^2 + a^2} \right)$$

$$= \log \left(\frac{s^2 + b^2}{s^2 + a^2} \right)^{1/2}$$

1b)

Given,

$$\vec{F} = (axy + z^3)\mathbf{i} + (3x^2 - 2)\mathbf{j} + (bx^2 - y)\mathbf{k}$$

Irrrotational, $\text{curl } \vec{F} = 0$

$$\text{curl } \vec{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} = 0$$

Here, $F_1 = axy + z^3$

$F_2 = 3x^2 - 2$

$F_3 = bx^2 - y$

$$\nabla \times \vec{F} = \mathbf{i} \left(\frac{\partial}{\partial y} (bx^2 - y) - \frac{\partial}{\partial z} (3x^2 - 2) \right) - \mathbf{j} \left(\frac{\partial}{\partial x} (3x^2 - 2) - \frac{\partial}{\partial z} (axy + z^3) \right) + \mathbf{k} \left(\frac{\partial}{\partial x} (axy + z^3) - \frac{\partial}{\partial y} (bx^2 - y) \right) = 0$$

$$\Rightarrow i((-1) + i) - j(6x - ay) + k(6x - ax) = 0$$

$$\Rightarrow 0 - 6xj + ayj + 6xk - axk = 0$$

$$\Rightarrow 6x(-j+k) + a(yj - xk) = 0$$

$$\Rightarrow 6x(-j+k) + a(yj - xk) = 0$$

$$\Rightarrow 6x + ay - ax = 0$$

$$\Rightarrow a(y-x) = -6x$$

$$\Rightarrow a = -6x - y + x$$

$$a = -5x - y$$

$$\nabla \times \vec{F} = i \left(\frac{\partial}{\partial y} (bx^2z^2 - y) - \frac{\partial}{\partial z} (3x^2 - 2) \right) - j \left(\frac{\partial}{\partial x} (bx^2z^2 - y) - \frac{\partial}{\partial z} (axy + z^3) \right) + k \left[\frac{\partial}{\partial x} (3x^2 - 2) - \frac{\partial}{\partial y} (axy + z^3) \right]$$

$$= i(-1 + i) - j(bz^2 - 3z^2) + k(6x - ax)$$

$$= -jz^2(b+3) + kx(6-a) = 0$$

$$b+3=0$$

$$b = -3$$

$$6-a=0$$

$$a = 6$$

1

IInd - INTERVAL

7) a) $f(x) = x^3 - 4x - 9$ b/w (2, 3)

then

$$f(2) = 2^3 - 4(2) - 9 = -9 < 0$$

$$f(3) = 3^3 - 4(3) - 9 = 6 > 0$$

$\therefore$ Root lies between 2 and 3

Now, we know that

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$x_1 = \frac{2(6) - 3(-9)}{6 + 9}$$

$$\Rightarrow x_1 = 2.600$$

then, $f(x_1) = f(2.600) = -1.824 < 0$

$\therefore$ Root lies between 2.600 and 3

Now,

$$a = 2.600 \quad \& \quad b = 3$$

$$f(a) = -1.824 \quad f(b) = 6$$

then

$$x_2 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{2.600(6) - 3(-1.824)}{6 + 1.824}$$

$$\Rightarrow x_2 = 2.693$$

$$f(x_2) = f(2.693) = -0.242 < 0$$

$\therefore$ Root lies b/w 2.693 and 3

Now,

$$a = 2.693 \quad b = 3$$

$$f(a) = -0.242 \quad f(b) = 6$$

then

$$x_3 = \frac{2.693(6) - 3(-0.242)}{6 + 0.242} = 2.705$$

$$f(x_3) = f(2.705) = -0.027 < 0$$

Root lies b/w 2.705 & 3

Now,

$$a = 2.705 \quad b = 3$$

$$f(a) = -0.027 \quad f(b) = 6$$

then,

$$x_4 = \frac{2.705(6) - 3(-0.027)}{6 + 0.027} = 2.706$$

$$f(x_4) = f(2.706) = -0.009 < 0$$

$\therefore$ Root lies b/w 2.706 & 3

Now, $a = 2.706$, $b = 3$

$$f(a) = -0.009 \quad f(b) = 6$$

then

$$x_5 = \frac{2.706(6) - 3(-0.009)}{6 + 0.009} = \underline{\underline{2.706}}$$

$\therefore$ Root lies b/w 2.706 & 3

7)
b)

x	$f(x)$	$f(x_0, x_1)$	$f(x_0, x_1, x_2)$	$f(x_0, x_1, x_2, x_3)$
4	1			
6	3	2.5	0.375	0
8	8		0.375	
10	16	4		

we know that

$$y = y_0 + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3)$$

$$y = 1 + (x-4)(1) + (x-4)(x-6)(0.375) + (x-4)(x-6)(x-8)(0)$$

$$y = 1 + x - 4 + (x-4)(x-6)\left(\frac{3}{8}\right)$$

$$y = 1 + x - 4 + x^2 - 6x - 4x + 24 \times \frac{3}{8}$$

$$y = 1 + x - 4 + x^2 - 10x + 9$$

$$y = 6 + x^2 - 9x$$

$$\therefore y = x^2 - 9x + 6$$

$$0 = \frac{0}{0.1} = f(x) \quad 0 = x$$

$$0.25 \cdot 0 = f(x) \quad 0.25 = x$$

$$1.28 \cdot 1 = f(x) \quad 1 = x$$

$$2.05 \cdot 1.5 = f(x) \quad 2.1 = x$$

$$3.08 \cdot 1.8 = f(x) \quad 3 = x$$

$$4.51 \cdot 2 = f(x) \quad 2.2 = x$$

$$6.03 \cdot 2.5 = f(x) \quad 3 = x$$

we know that

$$(x_0 + \epsilon_1 + \epsilon_2 + \epsilon_3) \delta + (x_0 + \epsilon_1 + \epsilon_2 + \epsilon_3) \epsilon + (x_0 + \epsilon_1 + \epsilon_2 + \epsilon_3) \epsilon + \frac{11\delta}{8} = 1$$

3) a)

$$f(x) = \frac{x}{\cos x}$$

given

$$[a, b] = [0, 3], m = 6$$

$$\text{then, } h = \frac{b-a}{n} = \frac{3-0}{6} = \frac{1}{2} \text{ or } 0.5$$

$$\text{we have } x: a = (x_0 = 0), (x_1 = 0.5), (x_2 = 1), (x_3 = 1.5), (x_4 = 2), (x_5 = 2.5), (x_6 = 3) = b$$

when

$$x_0 = 0, f(x_0) = \frac{0}{\cos 0} = 0$$

$$x_1 = 0.5, f(x_1) = 0.570$$

$$x_2 = 1, f(x_2) = 1.851$$

$$x_3 = 1.5, f(x_3) = 21.205$$

$$x_4 = 2, f(x_4) = -4.806$$

$$x_5 = 2.5, f(x_5) = -3.121$$

$$x_6 = 3, f(x_6) = -3.030$$

We know that,

$$I = \frac{3h}{8} + [(y_0 + y_n) + 3(y_1 + y_2 + y_4 + \dots + y_{n-1}) + 2(y_3 + y_6 + \dots + y_{n-2})]$$

$$I = \frac{3h}{8} [y_0 + y_6 + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)]$$

$$I = \frac{3(0.5)}{8} [0 + (-3.030) + 3(0.570 + 1.851 + (-4.806) + (-3.121)) + 2(21.205)]$$

$$I = 4.287$$

$\therefore$ By using Simpson's $3/8^{\text{th}}$ rule, $I = 4.287$

3) b)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
40	184			
		20		
50	204		2	
		22		0
60	226		2	
		24		0
70	250		2	
		26		0
80	276		2	
		28		
90	304			

$$n = \frac{x_n - x_0}{h}$$

$$n = \frac{90 - 40}{10}$$

$$\therefore n = 5$$

We know that

$$y(x) = y_0 + \Delta y_0 + \frac{\Delta^2 y_0}{2!} (x-x_0)^2 + \frac{\Delta^3 y_0}{3!} (x-x_0)^3$$

$$y(38) = 184 + 5(20) + \frac{5(x-x_0)^2}{2} (2) + \frac{5(x-x_0)(x-x_1)}{6} (0)$$

$$y(38) = 184 + 5(20) + \frac{5(38-40)^2}{2} (2)$$

$$y(38) = \underline{\underline{274}}$$

Final Answer = 274

$$0.5 - 0.1 = 0.4$$

$$0.4 - 0.1 = 0.3$$

0.1

$$0.3 - 0.1 = 0.2$$

0.5

0.3

0.1

0

0

0.5

0.3

0.1

0.5

0.3

0.1

0.5

0.3

0.1

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MOODLAKATTE INSTITUTE OF TECHNOLOGY

MOODLAKATTE-576 217, **KUNDAPURA** DIST: UDUPI

(Affiliated to VTU Belagavi and Approved by AICTE New Delhi)

ASSIGNMENT BOOK

ACADEMIC YEAR 20 22 -20 23

Department/Programme:	BSH/ECE		
Subject Title:	Engineering Mathematics	Subject Code:	BMATE201

Name: Deeksha Roll No.: 08

USN:

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 SEM: II Section: 'C'

FOR STAFF USE ONLY

Maximum Marks	Marks Obtained
20	10+10=20


Signature of
Student


Signature of
Subject Faculty in-charge


Signature of
HOD
Basis Science and Humanities
Moodlakatte Institute of Technology
Shri Shakhambhara Temple Road, Kundapura-576217
Udupi Dist Karnataka

ASSIGNMENT - 01

1. Find the workdone in moving a particle in the force field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + 2\hat{k}$ along the straight line from $(0,0,0)$ to $(2,1,3)$.

Solⁿ:

Here the path (curve) c along which the workdone is straight line from the origin $(0,0,0)$ to the point $(2,1,3)$ the cartesian of these straight line $x=y=z=t$ these equation of field $x=2t$ $y=t$ $z=3t$

$$\text{Here } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k} = 2t\hat{i} + t\hat{j} + 3t\hat{k}$$

$$\text{Given } \vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + 2\hat{k}$$

$$\frac{d\vec{r}}{dt} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\vec{F} = 3(2t)^2\hat{i} + (2 \cdot 2t \cdot 3t - t)\hat{j} + (3t)\hat{k}$$

$$\frac{d\vec{r}}{dt} = 2\hat{i} + \hat{j} + 3\hat{k}$$

$$\frac{d\vec{r}}{dt} = 2dt\hat{i} + dt\hat{j} + 3dt\hat{k}$$

$$\vec{F} \cdot d\vec{r} = (3(2t)^2 + (12t^2 - t)\hat{j} + 3t\hat{k}) \cdot (2dt\hat{i} + dt\hat{j} + 3dt\hat{k})$$

$$\vec{F} \cdot d\vec{r} = 24t^2 \cdot dt + (12t^2 - t) \cdot dt + 9t \cdot dt$$

We have a formula for workdone

$$\text{Workdone} = \int_c \vec{F} \cdot d\vec{r}$$

$$\therefore \text{Work done} = \int_C \vec{F} \cdot d\vec{r}$$

$$= \int_0^1 \{24t^2 + (12t^2 - t) + 9t\} \cdot dt$$

$$= \int_0^1 24t^2 + 12t^2 - t + 9t \cdot dt$$

$$= \int_0^1 36t^2 + 8t \cdot dt$$

$$= \left[36 \frac{t^3}{3} \right]_0^1 + \left[8 \frac{t^2}{2} \right]_0^1$$

$$= 12 [1 - 0] + 4 [1 - 0]$$

$$= 16.$$

$\therefore$ The workdone in moving a particle in the force.

field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the straight line from $(0,0,0)$ to $(2,1,3)$ is 16.

Q] Find the Laplace transform of the triangular wave function

$$f(t) = \begin{cases} t, & \text{if } 0 \leq t \leq a \\ 2a - t, & \text{if } a \leq t \leq 2a \end{cases}$$

Given: $f(t) = \begin{cases} t, & \text{if } 0 \leq t \leq a \\ 2a - t, & \text{if } a \leq t \leq 2a. \end{cases}$

We have Laplace transform of periodic function.
of period T formula.

$$\text{i.e. } L\{f(t)\} = \frac{1}{1 - e^{-st}} \int_0^T e^{-st} \cdot f(t) \cdot dt.$$

$$\begin{aligned} \text{Then } L\{f(t)\} &= \frac{1}{1 - e^{-st}} \left[\int_0^a e^{-st} \cdot t \cdot dt + \int_0^{2a} e^{-st} \cdot (2a - t) \cdot dt \right] \\ &= \frac{1}{1 - e^{-st}} \left[\int_0^a e^{-st} \cdot t \cdot dt + 2a \int_0^a e^{-st} \cdot dt - \int_0^a t \cdot e^{-st} \cdot dt \right] \quad \text{--- (1)} \end{aligned}$$

$$\text{Let us take } I = \int_I t \cdot e^{-st} \cdot dt.$$

$$\text{Then. } I = t \cdot \int e^{-st} \cdot dt - \int \left[\frac{d}{dt} (t) - \int e^{-st} \cdot dt \right] \cdot dt$$

$$I = t \cdot \frac{e^{-st}}{-s} - \int 1 - \frac{e^{-st}}{-s} \cdot dt.$$

$$I = -\frac{t \cdot e^{-st}}{s} + \frac{1}{s} \int e^{-st} \cdot dt$$

$$I = -\frac{t \cdot e^{-st}}{s} + \frac{1}{s} \cdot \frac{e^{-st}}{-s}$$

$$I = -\frac{t \cdot e^{-st}}{s} - \frac{e^{-st}}{-s^2}$$

Substitute I in (1).

Then (1) becomes,

$$\begin{aligned} \mathcal{L}\{f(t)\} &= \frac{1}{1-e^{-2as}} \left[\left(-\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2} \right) + 2a \left[\frac{e^{-st}}{-s} \right] - \left(-\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2} \right) \right]_0^{2a} \\ &= \frac{1}{1-e^{-2as}} \left\{ \left[-\frac{ae^{-sa}}{s} - \frac{e^{-sa}}{s^2} \right] - \left(\frac{0 \times e^{-s \times 0}}{s} - \frac{e^{-s \times 0}}{s^2} \right) + 2a \left[\frac{e^{-s(2a)}}{-s} \right] - \left(\frac{e^{-sa}}{-s} \right) - \left(-\frac{2ae^{-s2a}}{s} - \frac{e^{-s2a}}{s^2} \right) - \left(-\frac{ae^{-sa}}{s} - \frac{e^{-sa}}{s^2} \right) \right\} \end{aligned}$$

$$= \frac{1}{1-e^{-2as}} \left[-\frac{2ae^{-sa}}{s} + \frac{1}{s^2} + \frac{e^{-2as}}{s^2} \right]$$

$$= \frac{1}{1-e^{-2as}} \left[1 \cdot 2e^{-as} + e^{-2as} \right]$$

$$= \frac{1}{(1-e^{-2as})} \left[(1)^2 - 2 \cdot 1 \cdot e^{-as} + (e^{-as})^2 \right]$$

$$\mathcal{L}\{f(t)\} = \frac{1}{s^2(1-e^{-2as})} \cdot (1-e^{-as})^2$$

$$\mathcal{L}\{f(t)\} = \frac{1(1-e^{-as})(1-e^{-as})}{s^2(1+e^{-as})(1-e^{-as})} = \frac{1}{s^2} \left(\frac{1-e^{-as}}{1+e^{-as}} \right)$$

Multiply Numerator and Denominator by $e^{as/2}$

Then

$$\mathcal{L}\{f(t)\} = \left(\frac{1}{s^2} \right) \left(\frac{e^{as/2} - e^{-as} \times e^{as/2}}{e^{as/2} + e^{-as} \times e^{as/2}} \right)$$

$$\mathcal{L}\{f(t)\} = \frac{1}{s^2} \left(\frac{e^{as/2} - e^{-as/2}}{e^{as/2} + e^{-as/2}} \right)$$

$$\mathcal{L}\{f(t)\} = \frac{1}{s^2} \left(\frac{2 \sinh(as/2)}{2 \cosh(as/2)} \right)$$

Then

$$\mathcal{L}\{f(t)\} = \underline{\underline{\frac{1}{s^2} \tanh(as/2)}}$$

3. Evaluate $\text{curl}(\text{curl} \vec{F})$ and $\text{div}(\text{curl} \vec{F})$, if

$$\vec{F} = x^2y \hat{i} + y^2z \hat{j} + z^2x \hat{k}$$

Sol

$$\text{Given: } \vec{F} = x^2y \hat{i} + y^2z \hat{j} + z^2x \hat{k}$$

We have to evaluate $\text{curl}(\text{curl} \vec{F})$

$$\text{We have } \text{curl} \vec{F} = \nabla \times \vec{F}$$

$$\text{Then } \text{curl} \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & y^2z & z^2x \end{vmatrix}$$

$$\nabla \times \vec{F} = \hat{i} \left[\frac{\partial}{\partial y} (z^2x) - \frac{\partial}{\partial z} (y^2z) \right] - \hat{j} \left[\frac{\partial}{\partial x} (z^2x) - \frac{\partial}{\partial z} (x^2y) \right] + \hat{k} \left[\frac{\partial}{\partial x} (y^2z) - \frac{\partial}{\partial y} (x^2y) \right]$$

$$\text{Therefore } \text{curl} \vec{F} = \underline{\underline{-y^2 \hat{i} - z^2 \hat{j} - x^2 \hat{k}}}$$

Now we have to find $\text{curl}(\text{curl} \vec{F})$

$$\text{Then } \text{curl}(\text{curl} \vec{F}) = \nabla \times \text{curl} \vec{F}$$

$$\text{Then } \text{curl}(\text{curl} \vec{F}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y^2 & -z^2 & -x^2 \end{vmatrix}$$

$$\text{curl}(\text{curl} \vec{F}) = \hat{i} \left[\frac{\partial}{\partial y} (-x^2) - \frac{\partial}{\partial z} (-z^2) \right] - \hat{j} \left[\frac{\partial}{\partial x} (-x^2) - \frac{\partial}{\partial z} (-y^2) \right] + \hat{k} \left[\frac{\partial}{\partial x} (-x^2) - \frac{\partial}{\partial y} (-y^2) \right]$$

$$= \hat{i} [0 + 2z] - \hat{j} [-2x] + \hat{k} [0 + 2y]$$

$$\text{Curl}(\text{curl} \vec{F}) = \underline{2z\hat{i} + 2x\hat{j} + 2y\hat{k}}$$

We have to evaluate $\text{div}(\text{curl} \vec{F})$

$$\text{div}(\text{curl} \vec{F}) = \nabla \cdot \text{curl} \vec{F}$$

$$\text{We have } \text{curl} \vec{F} = -y^2\hat{i} - z^2\hat{j} - x^2\hat{k}$$

$$\text{Then } \nabla \cdot \text{curl} \vec{F} = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \cdot (-y^2\hat{i} - z^2\hat{j} - x^2\hat{k})$$

$$= \frac{\partial}{\partial x} (-y^2) + \frac{\partial}{\partial y} (-z^2) + \frac{\partial}{\partial z} (-x^2)$$

$$= \underline{\underline{\nabla \cdot \text{curl} \vec{F} = 0}}$$

$$\text{Therefore } \text{div}(\text{curl} \vec{F}) = \underline{\underline{0}}$$

4. Verify Stoke's theorem, for the vector field $\vec{F} = (2x-y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ over the upper half surface of $x^2+y^2+z^2=1$, bounded by its projection on the xy -plane.

Solⁿ

Given: $\vec{F} = (2x-y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, $x^2+y^2+z^2=1$

We have: Stoke's theorem is $\int_C \vec{f} \cdot d\vec{r} = \iiint_S (\nabla \times \vec{f}) \cdot \vec{n} \cdot d\vec{s}$

$$\text{LHS} = \int_C [(2x-y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}] \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= \int_C (2x-y)dx \quad \left[\text{Since } z=0 \text{ (upper half surface)} \right]$$

$$x^2+y^2=1.$$

$x = \cos\theta$, $y = \sin\theta$, $dx = -\sin\theta \cdot d\theta$
 θ varies from $0 \rightarrow 2\pi$

$$\int_0^{2\pi} (2\cos\theta - \sin\theta) (-\sin\theta d\theta)$$

$$\Rightarrow \int_0^{2\pi} -2\sin\theta \cos\theta \cdot d\theta + \int_0^{2\pi} \sin^2\theta \cdot d\theta.$$

$$= - \int_0^{2\pi} \sin 2\theta \cdot d\theta + \int_0^{2\pi} \frac{(1 - \cos 2\theta)}{2} \cdot d\theta.$$

$$+ \left[\frac{\cos 2\theta}{2} \right]_0^{2\pi} + \frac{1}{2} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{2\pi}$$

$$\Rightarrow \left[\frac{1}{2} - \frac{1}{2} \right] + \frac{1}{2} [(2\pi - 0) - (0 - 0)]$$

$$= \pi$$

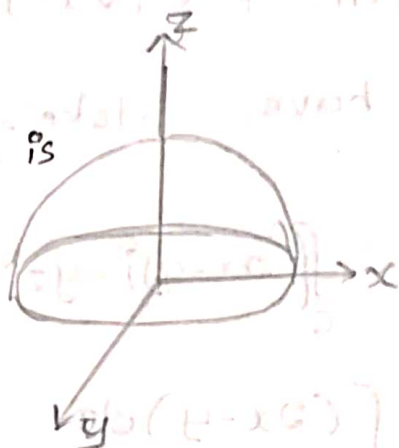
$$\therefore \text{LHS} = \underline{\underline{\pi}}$$

$$\text{RHS} = \iint_S (\nabla \times f) \cdot \vec{n} \cdot d\vec{s}$$

The projection of S on xy -plane is

$$x^2 + y^2 = 1 \text{ and } z = 0$$

$$\therefore d\vec{s} = \frac{dx dy}{|\vec{n} \cdot \hat{k}|}$$



$$\nabla \times f = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (2x-y) & -yz^2 & -y^2z \end{vmatrix}$$

$$= \hat{i} (-2yz + 2yz) - \hat{j} (0) + \hat{k} (0 - (-1))$$

$$= \hat{k}$$

$$\nabla \times f = \hat{k} \quad \phi = x^2 + y^2 + z^2 = 1$$

$$\nabla \phi = \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) (x^2 + y^2 + z^2 - 1)$$

$$= 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$

$$= 2(x\hat{i} + y\hat{j} + z\hat{k})$$

$$|\nabla \phi| = \sqrt{4x^2 + 4y^2 + 4z^2} = 2\sqrt{x^2 + y^2 + z^2} = \underline{\underline{2}} \quad (\because x^2 + y^2 + z^2 = 1)$$

$$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{2(x\hat{i} + y\hat{j} + z\hat{k})}{2}$$

$$= \underline{x\hat{i} + y\hat{j} + z\hat{k}}$$

$$\vec{n} \cdot \hat{k} = x\hat{i} + y\hat{j} + z\hat{k} \cdot \hat{k} = \underline{z}$$

$$\text{Now } \iint_S (\nabla \times f) \cdot \vec{n} \, ds = \iint_S z \cdot \frac{dxdy}{z}$$

$$= \iint_R dxdy = \pi \quad (\because R = \pi)$$

$$\text{RHS} = \pi$$

$$\underline{\text{LHS} = \text{RHS}}$$

$\therefore$ ~~Stoke's theorem~~ is verified.

5. Express $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$ in terms of the

Heaviside unit step function and hence find $L\{f(t)\}$

Solⁿ

We have

$$f(t) = f_1(t) + f_2(t) - f_1(t) \cup (t-a) \dots$$

By Heaviside shift theorem,

$$\text{Here } f(t) = \cos t + (\cos 2t - \cos t) \cup (t - \pi) + (\cos 3t - \cos 2t) \cup (t - 2\pi)$$

$$L\{f(t)\} = L\{\cos t\} + L\{\cos 2t - \cos t\} \cup (t - \pi) + L\{\cos 3t - \cos 2t\}$$

$$u(t-2\pi)$$

Also we know,

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as} \cdot f(s)$$

$$f(t-\pi) = \cos 2t - \cos t \quad (t = t+\pi)$$

$$f(t) = \cos 2(t+\pi) - \cos(t+\pi)$$

$$F(t) = \cos 2t + \cos t$$

$$G(t-2\pi) = \cos 3t - \cos 2t \quad (t = t+2\pi)$$

$$G(t) = \cos 3(t+2\pi) - \cos 2(t+2\pi)$$

$$G(t) = \cos 3t - \cos 2t$$

Now find $\mathcal{L}\{F(t)\}$

$$= \mathcal{L}\{\cos 2t + \cos t\}$$

$$F(s) = \frac{s}{s^2+4} + \frac{s}{s^2+1}$$

$\mathcal{L}\{G(t)\}$

$$\Rightarrow \mathcal{L}\{\cos 3t - \cos 2t\}$$

$$G(s) = \frac{s}{s^2+9} - \frac{s}{s^2+4}$$

By the formula,

$$\mathcal{L}\{f(t-\pi)u(t-\pi)\} = e^{-\pi s} \cdot f(s)$$

$$= e^{-\pi s} \left[\frac{s}{s^2+4} + \frac{s}{s^2+1} \right] \rightarrow \textcircled{1}$$

$$\mathcal{L}\{G(t-2\pi)u(t-2\pi)\} = e^{-2\pi s} \cdot G(s)$$

$$= e^{-2\pi s} \left[\frac{s}{s^2+9} - \frac{s}{s^2+4} \right] \rightarrow \textcircled{2}$$

$$\mathcal{L}\{f(t)\} = e^{-\pi s} \left[\frac{s}{s^2+4} + \frac{s}{s^2+1} \right] + e^{-2\pi s} \left[\frac{s}{s^2+9} - \frac{s}{s^2+4} \right] \quad (\text{adding } \textcircled{1} \text{ \& } \textcircled{2})$$

6. If $\vec{A} = 2x^2\hat{i} - 3y^2\hat{j} + xz^2\hat{k}$ and $\phi = 2x - x^3y$ compute $\vec{A} \cdot \nabla\phi$ and $\vec{A} \times \nabla\phi$ at $(1, -1, 1)$.

Solⁿ Given : $\vec{A} = 2x^2\hat{i} - 3y^2\hat{j} + xz^2\hat{k}$

$$\phi = 2x - x^3y$$

$$\nabla\phi = \left(\frac{\partial}{\partial x}\right)\hat{i} + \frac{\partial}{\partial y}(\hat{j}) + \frac{\partial}{\partial z}(\hat{k}) \cdot 2x - x^3y$$

$$\nabla\phi = (-3x^2y)\hat{i} + (-x^3)\hat{j} + 2(\hat{k})$$

$$\nabla\phi = (-3x^2y)\hat{i} + (-x^3)\hat{j} + 2(\hat{k})$$

$$\begin{aligned} \vec{A} \cdot \nabla\phi &= 2x^2\hat{i} - 3y^2\hat{j} + xz^2\hat{k} \cdot (-3x^2y)\hat{i} + (-x^3)\hat{j} + 2(\hat{k}) \\ &= -6x^4y + 3x^3yz + 2xz^2 \end{aligned}$$

$$\vec{A} \cdot \nabla\phi_{[1, -1, 1]} = 6 + (-3) + 2$$

$$\vec{A} \cdot \nabla\phi = 5$$

$$\therefore \underline{\vec{A} \cdot \nabla\phi = 5}$$

$$\vec{A} = 2x^2\hat{i} - 3y^2\hat{j} + xz^2\hat{k}$$

$$\Rightarrow \left. \vec{A} \right|_{(1, -1, 1)} = 2(1)^2\hat{i} - 3(-1)(1)\hat{j} + (1)(1)^2\hat{k}$$

$$\vec{A} = 2\hat{i} + 3\hat{j} + 1\hat{k} \quad \text{--- (1)}$$

We have

$$\nabla\phi = 3\hat{i} - 1\hat{j} + 2\hat{k} \quad \text{--- (2)}$$

(1, -1, 1)

By (1) & (2)

$$\begin{aligned} \vec{A} \times \nabla\phi &= \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 3 & -1 & -2 \end{vmatrix} \\ &= \hat{i}[6+1] - \hat{j}[4-3] + \hat{k}[-2-9] \\ &= \underline{\underline{7\hat{i} - \hat{j} - 11\hat{k}}} \end{aligned}$$

$$\therefore \vec{A} \times \nabla\phi = \underline{\underline{7\hat{i} - \hat{j} - 11\hat{k}}}$$

7. Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$

Let $v = x^3 + y^3 + z^3 - 3xyz$.

then

$\vec{F}$ becomes.

$$\vec{F} = \nabla v$$

$$\vec{F} = \hat{i} \frac{\partial v}{\partial x} + \hat{j} \frac{\partial v}{\partial y} + \hat{k} \frac{\partial v}{\partial z}$$

$$\vec{F} = \hat{i} \frac{\partial}{\partial x} (x^3 + y^3 + z^3 - 3xy) + \hat{j} \frac{\partial}{\partial y} (x^3 + y^3 + z^3 - 3xy) + \hat{k} \frac{\partial}{\partial z} (x^3 + y^3 + z^3 - 3xy)$$

$$\vec{F} = \hat{i} (3x^2 - 3yz) + \hat{j} (3y^2 - 3xz) + \hat{k} (3z^2 - 3xy)$$

$$\vec{F} = \hat{i} F_x + \hat{j} F_y + \hat{k} F_z$$

Here $F_x = 3x^2 - 3yz$ $F_y = 3y^2 - 3xz$ $F_z = 3z^2 - 3xy$

then

$$\text{div } \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\Rightarrow \frac{\partial}{\partial x} (3x^2 - 3yz) + \frac{\partial}{\partial y} (3y^2 - 3xz) + \frac{\partial}{\partial z} (3z^2 - 3xy)$$

$$\text{div } \vec{F} = 6x + 6y + 6z$$

$$\therefore \text{div } \vec{F} = \underline{6(x+y+z)}$$

then. $\text{curl } \vec{F} = \hat{i} \left(\frac{\partial}{\partial y} F_z - \frac{\partial}{\partial z} F_y \right) - \hat{j} \left(\frac{\partial}{\partial x} F_z - \frac{\partial}{\partial z} F_x \right)$
 $+ \hat{k} \left(\frac{\partial}{\partial x} F_y - \frac{\partial}{\partial y} F_x \right)$

$$\text{curl } \vec{F} = \hat{i} \left(\frac{\partial}{\partial y} (3z^2 - 3xy) - \frac{\partial}{\partial z} (3y^2 - 3xz) \right) - \hat{j} \left(\frac{\partial}{\partial x} (3z^2 - 3xy) - \frac{\partial}{\partial z} (3x^2 - 3yz) \right) + \hat{k} \left(\frac{\partial}{\partial x} (3y^2 - 3xz) - \frac{\partial}{\partial y} (3x^2 - 3yz) \right)$$

$$\text{curl } \vec{F} = \hat{i} (-3x + 3x) - \hat{j} (-3y + 3y) + \hat{k} (-3z + 3z)$$

$$\therefore \underline{\underline{\text{curl } \vec{F} = 0}}$$

8. If $\phi = 2x^3y^2z^4$ find $\text{div}(\text{grad } \phi)$ and verify that $\nabla \cdot \nabla \phi = \nabla^2 \phi$.

Solⁿ

$$\phi = 2x^3y^2z^4$$

$$\text{grad } \phi = \nabla \phi = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \cdot 2x^3y^2z^4$$

$$\nabla \phi = (6x^2y^2z^4)\hat{i} + (4x^3yz^4)\hat{j} + (8x^3y^2z^3)\hat{k}$$

$$\text{div}(\text{grad } \phi) = \nabla \cdot \nabla \phi$$

$$= \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \cdot (6x^2y^2z^4)\hat{i} + (4x^3yz^4)\hat{j} + (8x^3y^2z^3)\hat{k}$$

$$= \frac{\partial}{\partial x} (6x^2y^2z^4) + \frac{\partial}{\partial y} (4x^3yz^4) + \frac{\partial}{\partial z} (8x^3y^2z^3)$$

$$= 12xy^2z^4 + 4x^3z^4 + 24x^3y^2z^2 \quad \text{--- (1)}$$

Then to find $\nabla^2 \phi$

$$\nabla^2 \phi = \left(\frac{\partial^2}{\partial x^2} \right) \hat{i} + \left(\frac{\partial^2}{\partial y^2} \right) \hat{j} + \left(\frac{\partial^2}{\partial z^2} \right) \hat{k} \cdot 2x^3y^2z^4$$

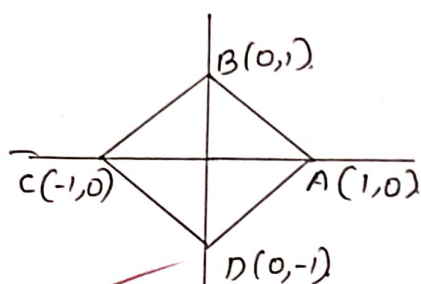
$$\nabla^2 \phi = \frac{\partial}{\partial x} (6x^2y^2z^4) + \frac{\partial}{\partial y} (4x^3yz^4) + \frac{\partial}{\partial z} (8x^3y^2z^3)$$

$$\nabla^2 \phi = 12xy^2z^4 + 4x^3z^4 + 24x^3y^2z^2 \rightarrow \textcircled{2}$$

from ① and ③

$$\nabla \cdot \nabla \phi = \nabla^2 \phi = 12xy^2z^4 + 4x^3z^4 + 24x^3y^2z^2.$$

9. Evaluate $\int_C xy dx + xy^2 dy$ by Stoke's theorem where C is the square in the xy -plane with vertices $(1,0)$, $(-1,0)$, $(0,1)$, $(0,-1)$.



We have Stoke's theorem $\int_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl } \vec{F} \cdot d\vec{s}$.

From the given it is evident that

$$\vec{F} = xy\hat{i} + xy^2\hat{j}.$$

$$\text{Curl } \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & xy^2 & 0 \end{vmatrix}$$

$$\text{curl } \vec{F} = (y^2 - x)\hat{k}$$

$$\text{Then } \iint_S \text{curl } \vec{F} \cdot d\vec{s} = \iint_S (y^2 - x) \cdot dx dy$$

It can be clearly seen from the figure that x varies from -1 to 1 y varies from -1 to 1 .

$$\text{Now } \iint_S \text{curl } \vec{F} \cdot d\vec{s}$$

$$= \int_{x=-1}^1 \int_{y=-1}^1 (y^2 - x) dy dx$$

$$= \int_{x=-1}^1 \left[\frac{y^3}{3} - xy \right]_{y=-1}^1 dx$$

$$= \int_{-1}^1 \left(\frac{2}{3} - 2x \right) dx$$

$$= \left[\frac{2}{3}x - x^2 \right]_{-1}^1$$

$$= \frac{2}{3}(1+1) - (1-1)$$

$$= \frac{4}{3}$$

Thus

$$\int_C xy dx + x^2 dy = \underline{\underline{4/3}}$$

10 Find the Laplace transform of the periodic function

$$f(t) = \begin{cases} t, & 0 < t < \pi \\ \pi - t, & \pi < t < 2\pi \end{cases}$$

Here $T = 2\pi$.

solⁿ

WKT $\mathcal{L}\{f(t)\} = \frac{1}{1 - e^{-st}} \int_0^T e^{-st} f(t) \cdot dt$

$$= \frac{1}{1 - e^{-2s\pi}} \int_0^{2\pi} e^{-st} f(t) \cdot dt$$

$$= \frac{1}{1 - e^{-2s\pi}} \left[\int_0^{\pi} e^{-st} t \, dt + \int_{\pi}^{2\pi} e^{-st} (2\pi - t) \, dt \right]$$

$$= \frac{1}{1 - e^{-2\pi s}} \left[\left\{ t \frac{e^{-st}}{s} + \frac{e^{-st}}{s^2} \right\} \right]_0^{\pi} + \left[(2\pi - t) \frac{e^{-st}}{-s} - \frac{(-1) e^{-st}}{(-s)^2} \right]_{\pi}^{2\pi}$$

$$= \frac{1}{1 - e^{-2\pi s}} \left[\left\{ \frac{\pi e^{-s\pi}}{-s} - \frac{e^{-s\pi}}{s^2} \right\} - \left[0 - \frac{e^0}{s^2} \right] + \left[e^{-2\pi} - 2\pi \right] \right]$$

$$= \frac{1}{1 - e^{-2\pi s}} \left[\frac{\pi e^{-s\pi}}{-s} - \frac{e^{-s\pi}}{s^2} \right] + \left[2\pi - \frac{e^{-2\pi s}}{-s} + \frac{e^{-2\pi s}}{s^2} \right]$$

$$+ \left[(2\pi - \pi) \frac{e^{-s\pi}}{-s} + \frac{e^{-s\pi}}{s^2} \right]$$

$$= \frac{1}{1 - e^{-2\pi s}} \left[\frac{-\pi e^{-s\pi}}{s} - \frac{e^{-s\pi}}{s^2} + \frac{1}{s^2} + \frac{e^{-2\pi s}}{s^2} + \frac{\pi e^{-2s}}{s} - \frac{e^{-s\pi}}{s^2} \right]$$

$$\mathcal{L}\{f(t)\} = \frac{1}{s^2} \left[1 - 2e^{-\pi s} + (e^{-\pi s})^2 \right]$$

$$1 - (e^{-\pi s})^2$$

$$= \frac{1}{s^2} \frac{[1 - e^{-\pi s}]}{[1 + e^{-\pi s}]}$$

$$= \frac{1}{s^2} \left[\frac{e^{\pi s/2} - e^{-\pi s/2}}{e^{\pi s/2} + e^{-\pi s/2}} \right]$$

$$= \frac{1}{s^2} \tanh\left(\frac{\pi s}{2}\right)$$

10/10

MODULE - 3

MODEL QUESTION PAPER - 2

ASSIGNMENT - 2

1. Find the Laplace transform of i] $t e^{-t} \sin 4t$ ii] $\frac{1 - \cos at}{t}$

Here $e^{-t} \sin 4t = f(t)$.

To find $L\{e^{-t} \sin 4t\} = L\{e^{at} \cdot f(t)\}$

$$\text{Now } L\{f(t)\} = L\{\sin 4t\} = \frac{4}{s^2 + 16}$$

$$\text{Now } L\{e^{-t} \sin 4t\} = \left(\frac{4}{s^2 + 16}\right)_{s \rightarrow s+1}$$

$$= \frac{4}{(s+1)^2 + 16} = f(s)$$

$$\text{Now to find } L\{t e^{-t} \sin 4t\} = (-1) \frac{d}{ds} \left(\frac{4}{(s+1)^2 + 16} \right)$$

$$= - \left[\frac{-4 \cdot 2(s+1)}{[(s+1)^2 + 16]^2} \right]$$

$$= \frac{8(s+1)}{[(s+1)^2 + 16]^2} = \left[\frac{8[s+2]}{s^2 + 1s + 16} \right]$$

ii] $\frac{1 - \cos at}{t}$

$$= f(t) = 1 - \cos at \quad \therefore f[f(t)] = f(s)$$

$$= f(1 - \cos at) = L\{1\} - L\{\cos at\}$$

$$= \frac{1}{s} - \frac{s}{s^2 + a^2}$$

$$\begin{aligned}
 L\left[\frac{1-\cos at}{t}\right] &= \int_s^\infty \left(\frac{1}{u} - \frac{u}{u^2+a^2}\right) \cdot du - \int_s^\infty \frac{1}{u} du - \int_s^\infty \frac{u}{u^2+a^2} \cdot du \\
 &= \ln u \int_s^\infty -\frac{1}{2} \ln(u^2+a^2) \int_s^\infty \ln u = \frac{1}{2} \ln u^2 \\
 &\quad - \frac{1}{2} \ln u^2 \int_s^\infty -\frac{1}{2} \ln(u^2+a^2) \int_s^\infty \ln \left(\frac{u}{u^2+a^2}\right) = \ln u \ln u \\
 &= \frac{1}{2} \ln\left(\frac{u^2}{u^2+a^2}\right) \Big|_s^\infty
 \end{aligned}$$

$$\begin{aligned}
 L\left[\frac{1-\cos at}{t}\right] &= \int_s^\infty \left(\frac{1}{u} - \frac{u}{u^2+a^2}\right) \cdot du = \frac{1}{2} \ln\left(\frac{u^2}{u^2+a^2}\right) \Big|_s^\infty \\
 &= \frac{1}{2} \ln\left(\frac{1}{1+a^2/u^2}\right) \Big|_s^\infty = \frac{1}{2} \ln\left(\frac{1}{1+\frac{a^2}{s^2}}\right) - \frac{1}{2} \ln\left(\frac{1}{1+\frac{a^2}{s^2}}\right) \\
 &= \frac{1}{2} \ln(1) - \frac{1}{2} \ln\left(\frac{s^2}{s^2+a^2}\right) \\
 &= -\frac{1}{2} \ln\left(\frac{s^2}{s^2+a^2}\right) = \frac{1}{2} \ln\left(\frac{s^2+a^2}{s^2}\right)
 \end{aligned}$$

2] Find the Laplace transform of the square wave function of t period $2a$, define by $f(t) = \begin{cases} k & 0 < t < a \\ -k & a < t < 2a \end{cases}$

Solⁿ:

Here $T = 2a$

WKT $L\{f(t)\} = \frac{1}{1-e^{-st}} \int_0^T e^{-st} \cdot f(t) \cdot dt$

Now $L\{f(t)\} = \frac{1}{1-e^{-2as}} \left[\int_0^a e^{-st} \cdot k dt + \int_a^{2a} e^{-st} (-k) dt \right]$

$$= \frac{1}{1-e^{-2as}} \left[k \left(\frac{e^{-st}}{-s}\right)_0^a - k \left(\frac{e^{-st}}{s}\right)_a^{2a} \right]$$

$$= \frac{1}{1-e^{-2as}} \left[\frac{-k}{s} (e^{-as} - e^0) + \frac{k}{s} (e^{-2as} - e^{-as}) \right]$$

$$= \frac{1}{1-e^{-2as}} \left[\frac{-k}{s} (e^{-as} - 1) + \frac{k}{s} (e^{-2as} - e^{-as}) \right]$$

$$= \frac{k}{s(1-e^{-2as})} \left[e^{-2as} - 2e^{-as} + 1 \right]$$

$$= \frac{k}{s(1-e^{-2as})} \left[e^{-2as} - 2e^{-as} + 1 \right]$$

$$= \frac{k(1-e^{-2as})^2}{s(1-e^{-2as})} = \frac{k(1-e^{-as})(1-e^{-as})}{s(1-e^{-as})(1-e^{-as})}$$

$$= \frac{k}{s} \left(\frac{1-e^{-as}}{1+e^{-as}} \right) \times \frac{e^{as/2}}{e^{as/2}}$$

$$= \frac{k}{s} \left(\frac{e^{as/2} - e^{-as/2}}{e^{as/2} + e^{-as/2}} \right) = \underline{\underline{\frac{k}{s} \tanh(as/2)}}$$

3] Express $f(t) = \begin{cases} \cos t & 0 < t < \pi \\ \cos 2t & \pi < t < 2\pi \\ \cos 3t & t > 2\pi \end{cases}$ in terms of the Heaviside function unit & hence find $\mathcal{L}\{f(t)\}$

Solⁿ $f(t) = \cos t + (\cos 2t - \cos t)u(t-\pi) + (\cos 3t - \cos 2t)u(t-2\pi)$

by using property

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{\cos t\} + \mathcal{L}\{(\cos 2t - \cos t)u(t-\pi)\} + \mathcal{L}\{(\cos 3t - \cos 2t)u(t-2\pi)\} \quad \text{--- (1)}$$

$$\Rightarrow \mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}f(s)$$

$$= F(t-\pi) = \cos 2t - \cos t$$

$$= F(t) = \cos 2(t+\pi) - \cos(t+\pi)$$

$$F(t) = \cos 2t + \cos t \quad \text{--- (2)}$$

$$G(t-2\pi) = \cos 3t - \cos 2t$$

$$t = t+2\pi$$

$$G(t) = \cos 3(t+2\pi) - \cos 2(t+2\pi)$$

$$G(t) = \cos 3t - \cos 2t \quad \text{--- (3)}$$

Apply Laplace on both equation (2) & (3)

$$F(s) = \frac{s}{s^2+4} + \frac{s}{s^2+1}$$

$$G(s) = \frac{s}{s^2+9} - \frac{s}{s^2+4}$$

$$\mathcal{L}[F(t-\pi)u(t-\pi)] = e^{-\pi s} F(s)$$

$$= e^{-\pi s} \left[\frac{s}{s^2+4} + \frac{s}{s^2+1} \right]$$

$$\mathcal{L}[G(t-2\pi)u(t-2\pi)] = e^{-2\pi s} G(s)$$

$$= e^{-2\pi s} \left[\frac{s}{s^2+9} - \frac{s}{s^2+4} \right]$$

Here eqⁿ ① becomes.

$$\mathcal{L}\{f(t)\} = \frac{s}{s^2+1} + e^{-\pi s} \left[\frac{s}{s^2+4} + \frac{s}{s^2+1} \right] + e^{-2\pi s}$$

$$= \left[\frac{s}{s^2+9} - \frac{s}{s^2+4} \right]$$

4 Find $\mathcal{L}^{-1} \left\{ \frac{2s^2-6s+5}{s^3-6s^2+11s-6} \right\}$

Solⁿ

Given $f(s) = \frac{2s^2-6s+5}{s^3-6s^2+11s-6} = \frac{2s^2-6s+5}{(s-1)(s-2)(s-3)}$

using partial fraction.

$$\frac{2s^2-6s+5}{(s-1)(s-2)(s-3)} = \frac{A}{s-1} + \frac{B}{s-2} + \frac{C}{s-3}$$

$$= A(s-2)(s-3) + B(s-3)(s-1) + C(s-1)(s-2)$$

put $s=1$

$$2-6+5 = A(-1)(-2)$$

$$1 = 2A \Rightarrow A = 1/2$$

put $s=2$

$$8-12+5 = B(1)(-1)$$

$$1 = -B \Rightarrow B = -1$$

put $s=3$

$$18-18+5 = C(2)(1)$$

$$C = 5/2$$

$$\therefore f(s) = \frac{1}{2} \left(\frac{1}{s-1} \right) - \left(\frac{1}{s-2} \right) + \frac{5}{2} \left(\frac{1}{s-3} \right)$$

Taking inverse laplace transform on b.s

$$= \mathcal{L}^{-1} \{ f(s) \} = \frac{1}{2} \mathcal{L}^{-1} \left(\frac{1}{s-1} \right) - \mathcal{L}^{-1} \left(\frac{1}{s-2} \right) + \frac{5}{2} \mathcal{L}^{-1} \left(\frac{1}{s-3} \right)$$

$$= \mathcal{L}^{-1} \{ f(s) \} = \frac{1}{2} e^t - e^{2t} + \frac{5}{2} e^{3t}$$

5 Find $\mathcal{L}^{-1} \left\{ \frac{1}{s^3(s^2+1)} \right\}$ using the convolution theorem.

Solⁿ

According to the equation:

$$\mathcal{L}^{-1} \left[\frac{1}{s^3(s^2+1)} \right] = \mathcal{L}^{-1} \left[\left(\frac{1}{s^3} \right) * \left(\frac{1}{s^2+1} \right) \right]$$

Let

$$f(s) = \frac{1}{s^3}, \quad g(s) = \frac{1}{s^2+1}$$

$$\text{Now } f(s) = \mathcal{L}^{-1} \left[F(s) \right] = \mathcal{L}^{-1} \left[\frac{1}{s^3} \right]$$

$$g(s) = \mathcal{L}^{-1} \left[G(s) \right] = \mathcal{L}^{-1} \left[\frac{1}{s^2+1} \right]$$

By convolution theorem:

$$\mathcal{L}^{-1} [F(s) * G(s)] = \int_0^A f(u) \cdot g(A-u) \cdot du$$

$$= \int_0^A \left(\frac{u^2}{2} \right) \cdot (\sin(A-u)) \cdot du$$

$$= \frac{1}{2} \int (u^2) \left(\frac{-\cos(t-u)}{-1} \right) \cdot \left(\frac{\sin(t-u)}{-1} \right)$$

$$(2) \left(\frac{\cos(t-u)}{(-1)(-1)} - \cos \right) \Big|_0^t$$

$$\frac{2 + 2t + t^2 + t^2 + 1}{1+2} = \frac{[t^2 + 2t + 3]}{3}$$

Integration by parts.

$$= \frac{1}{2} \left[u^2 \cos(t-u) + 2u \sin(t-u) - 2 \cos(t-u) \right]_0^t$$

$$= \frac{1}{2} \left[[t^2 \cos(0) + 2t \sin(0) - 2 \cos(0)] - [0 + 0 - 2 \cos(A)] \right]$$

$$= \frac{1}{2} [t^2 + 0 - 2 + 2 \cos t]$$

$$= \frac{1}{2} [t^2 + 2 \cos t - 2]$$

6. Solve by Laplace transform method: $y'' + 4y' + 3y = e^{-t}$
given $y(0) = y'(0) = 1$.

Solⁿ

Linear equation is $y''(t) + 4y'(t) + 3y(t) = e^{-t}$.

Then.

$$\mathcal{L}\{y''(t)\} + 4\mathcal{L}\{y'(t)\} + 3\mathcal{L}\{y(t)\} = \mathcal{L}\{e^{-t}\}$$

$$\Rightarrow [s^2 \mathcal{L}\{y(t)\} - s y(0) - y'(0)] + 4[s \mathcal{L}\{y(t)\} - y(0)] + 3\mathcal{L}\{y(t)\} = \frac{1}{s+1}$$

$$= \frac{1}{s+1}$$

$$= \mathcal{L}\{y(t)\} [s^2 + 4s + 3] - s - 1 - 4 = \frac{1}{s+1}$$

$$= \mathcal{L}\{y(t)\} [s^2 + 4s + 3] - s - 5 = \frac{1}{s+1}$$

$$\mathcal{L}\{y(t)\} [s^2 + 4s + 3] = \frac{1}{s+1} + s + 5$$

$$\mathcal{L}\{y(t)\} [s^2 + 4s + 3] = \frac{1 + s(s+1) + 5(s+1)}{s+1}$$

$$\mathcal{L}\{y(t)\} [s^2 + 4s + 3] = \frac{1 + s^2 + s + 5s + 5}{s+1}$$

$$\mathcal{L}\{y(t)\} [s^2 + 4s + 3] = \frac{s^2 + 6s + 6}{s+1}$$

$$\mathcal{L}\{f(t)\} = \frac{s^2 + 6s + 6}{(s+1)[s^2 + 4s + 3]}$$

$$y(t) = \mathcal{L}^{-1} \left\{ \frac{s^2 + 6s + 6}{(s+1)(s^2 + 4s + 3)} \right\}$$

$$\begin{aligned} \text{Now } (s^2 + 4s + 3) &= s^2 + 3s + 1s + 3 \\ &= s(s+3) + 1(s+3) \\ &= (s+3)(s+1) \end{aligned}$$

$$\therefore y(t) = \mathcal{L}^{-1} \left\{ \frac{s^2 + 6s + 6}{(s+1)^2(s+3)} \right\}$$

MODULE - QUESTION PAPER - 1. MODULE - 3.

1. Find the Laplace transform of i] $e^{-3t}(2\cos 5t - 3\sin 5t)$

i] $e^{-3t}(2\cos 5t - 3\sin 5t)$

ii] $\frac{\cos at - \cos bt}{t}$

$$\Rightarrow \mathcal{L}\{2\cos 5t - 3\sin 5t\} = 2\mathcal{L}\{\cos 5t\} - 3\mathcal{L}\{\sin 5t\}$$

$$= \frac{2s}{s^2 + 5^2} - \frac{15}{s^2 + 5^2} = \frac{2s - 15}{s^2 + 5^2}$$

($\because$ By shift property)

$$= \mathcal{L}\{e^{-3t}(2\cos 5t - 3\sin 5t)\} = \frac{2(s+3) - 15}{(s+3)^2 + 15^2}$$

$$= \frac{2s + 6 - 15}{s^2 + 6s + 9 + 25} = \frac{2s - 9}{s^2 + 6s + 35}$$

ii) $\frac{\cos at - \cos bt}{t}$

Soln

$$\mathcal{L}\{\cos at - \cos bt\} = \left(\frac{s}{s^2+a^2}\right) - \left(\frac{s}{s^2+b^2}\right)$$

$$\mathcal{L}\left\{\frac{\cos at - \cos bt}{t}\right\} = \int_s^\infty \left(\frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}\right) ds$$

$$= \left[\frac{1}{2} \log(s^2+a^2) - \frac{1}{2} \log(s^2+b^2) \right]_s^\infty$$

$$= \frac{1}{2} \left[\log\left(\frac{s^2+a^2}{s^2+b^2}\right) \right]_s^\infty$$

$$= \frac{1}{2} \left[\lim_{s \rightarrow \infty} \log\left(\frac{s^2+a^2}{s^2+b^2}\right) - \log\left(\frac{s^2+a^2}{s^2+b^2}\right) \right]$$

$$= \frac{1}{2} \left[\lim_{s \rightarrow \infty} \log\left(\frac{s^2+a^2}{s^2+b^2}\right) - \log\left(\frac{s^2+a^2}{s^2+b^2}\right) \right]$$

$$= \frac{1}{2} \left[\lim_{s \rightarrow \infty} \log\left(\frac{s^2(1+a^2/s^2)}{s^2(1+b^2/s^2)}\right) - \log\left(\frac{s^2+a^2}{s^2+b^2}\right) \right]$$

$$= \frac{1}{2} \left[\log\left(\frac{1+a^2/s^2}{1+b^2/s^2}\right) - \log\left(\frac{s^2+a^2}{s^2+b^2}\right) \right]$$

$$= \frac{1}{2} \left[\log 1 - \log\left(\frac{s^2+a^2}{s^2+b^2}\right) \right] = -\frac{1}{2} \log\left(\frac{s^2+a^2}{s^2+b^2}\right)$$

$$= \log\left(\frac{s^2+b^2}{s^2+a^2}\right)^{1/2}$$

$$= \log\left[\frac{s^2+b^2}{s^2+a^2}\right]^{1/2}$$

2. Find the Laplace transform of the triangular wave function.

$$f(t) = \begin{cases} t & \text{if } 0 \leq t \leq a \\ 2a-t & \text{if } a \leq t \leq 2a \end{cases}$$

Solⁿ

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-2as}} \int_0^{2a} e^{-st} f(t) dt$$

$$= \frac{1}{1-e^{-2as}} \left[\int_0^a e^{-st} t dt + \int_a^{2a} e^{-st} (2a-t) dt \right]$$

$$= \frac{1}{1-e^{-2as}} \left[\left[\frac{t e^{-st}}{-s} + \frac{e^{-st}}{s^2} \right]_0^a - \frac{e^{-st}}{s^2} \Big|_a^{2a} + \left[(2a-t) \frac{e^{-st}}{-s} \right]_a^{2a} - (-1) \frac{e^{-st}}{s^2} \Big|_a^{2a} \right]$$

$$= \frac{1}{1-e^{-2as}} \left[\frac{a e^{-sa}}{-s} - \frac{e^{-sa}}{s^2} + \frac{-a e^{-sa}}{s} + \frac{e^{-2as} - e^{-as}}{s^2} \right]$$

$$= \frac{1}{1-e^{-2as}} \left[\frac{-a e^{-as}}{s} - \frac{e^{-as}}{s^2} - \frac{1}{s^2} + \frac{a e^{-sa}}{s} + \frac{e^{-2as}}{s^2} - \frac{e^{-as}}{s^2} \right]$$

$$= \frac{1}{s^2(1-e^{-2as})} \left[1 - 2e^{-as} + e^{-2as} \right]$$

$$= \frac{1}{s^2(1-e^{-as})} \left[1^2 - 2e^{-as} + (e^{-as})^2 \right]$$

$$= \frac{1}{s^2(1+e^{-2as})(1-e^{-as})}$$

$$\frac{1(1-e^{-as})}{s^2(1+e^{-as})} = \frac{1}{s^2} \tanh \frac{as}{2}$$

4. Find the Laplace inverse transform of

i) $\frac{s}{s^2 + 2s + 3}$

Solⁿ $\frac{s}{s^2 + 2s + 3} = \frac{s}{(s^2 + 2s + 1) + 2}$

$$= \frac{s}{(s+1)^2 + 2^2}$$

Now $L^{-1} \left\{ \frac{s}{(s+1)^2 + 2^2} \right\} e^{+t} = L^{-1} \left\{ \frac{s}{s^2 + 2^2} \right\}$

$$= e^t \cdot L^{-1} \left\{ \frac{2s}{2(s^2 + 2^2)} \right\} = e^t \cdot \frac{1}{2} \cos 2t$$

ii) $\frac{1}{(s+2)^2}$

Solⁿ Given $= \frac{1}{(s+2)^2} = f(s)$

$$L^{-1} \{ f(s) \} = f(t)$$

$$= L^{-1} \left\{ \frac{1}{(s-a)^n} \right\} = \frac{t^{n-1}}{(n-1)!} \quad (\because \text{Formula})$$

here $n=2$ $a=-2$

$\therefore$ no. substitute n and a we get

$$f(t) = \frac{t^{2-1}}{(2-1)!} \cdot e^{-2t} = t e^{-2t}$$

$$= L^{-1} \left\{ \frac{1}{(s+2)^2} \right\} = \underline{t e^{-2t}}$$

5] using the convolution theorem find the inverse Laplace transform of $\frac{s}{(s^2 + a^2)^2}$

Solⁿ

Given: $\frac{s}{(s^2 + a^2)^2}$

$$= f(t) = \mathcal{L}^{-1} [F(s)]$$

$$g(t) = \mathcal{L}^{-1} [G(s)]$$

$$\text{Let } F(s) = \frac{1}{s^2 + a^2}$$

$$G(s) = \frac{s}{s^2 + a^2}$$

$$f(t) = \frac{\sin at}{a}$$

$$g(t) = \cos at$$

We have convolution theorem:

$$\mathcal{L}^{-1} [F(s) \cdot G(s)] = \int_{u=0}^t f(u) g(t-u) du$$

$$\mathcal{L}^{-1} \left[\frac{s}{(s^2 + a^2)^2} \right] = \int_{u=0}^t \frac{\sin au}{a} \cdot \cos (at - au) \cdot du$$

$$= \frac{1}{a} \int_{u=0}^t \frac{1}{2} [\sin (au + at - au) + \sin (au - at + au)] du$$

$$= \frac{1}{2a} \int_{u=0}^t [\sin at + \sin (2au - at)] du$$

$$= \frac{1}{2a} \left\{ \sin at [u]_0^t - \left[\frac{\cos (2au - at)}{2a} \right]_{u=0}^t \right\}$$

$$= \frac{1}{2a} \left\{ \sin at (t - 0) - \frac{1}{2a} (\cos at - \cos at) \right\}$$

$$\mathcal{L}^{-1} \left[\frac{s}{(s^2 + a^2)^2} \right] = \frac{t \cdot \sin at}{2a}$$

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14/09/23

